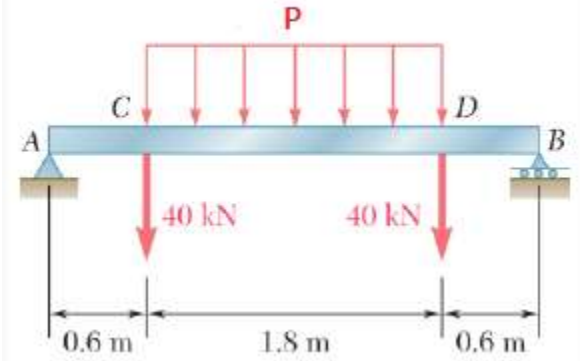
Homework # 5 - Chapter 5
EGR 2312-01, Fall 2019

Name: …… **Instructor: Dr. A. Bentamy**
Due Date:

Problem #1:

Consider the given beam and loading where $P = 39$ kN/m.



From the figures given below, identify the correct bending-moment diagram and determine the maximum absolute value of the bending moment. (Round the final answer to two decimal places.)

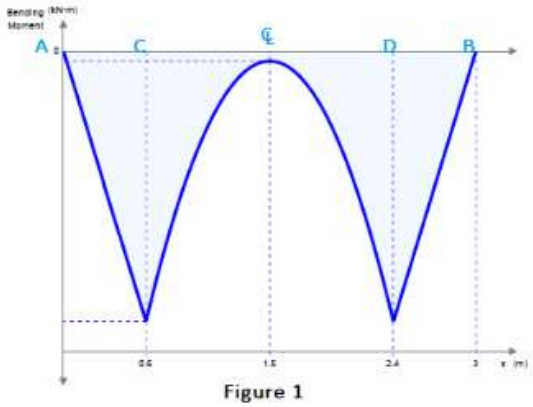


Figure 1

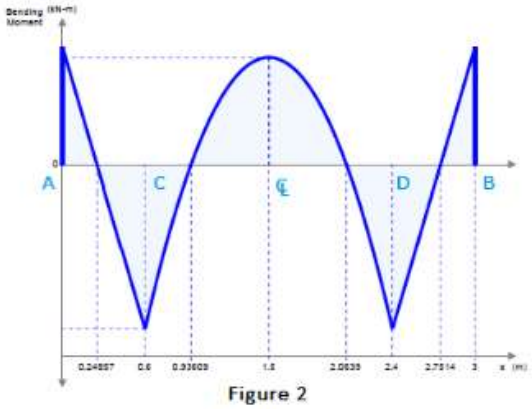


Figure 2

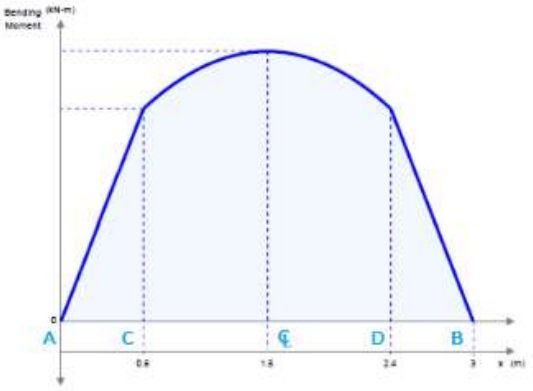


Figure 3

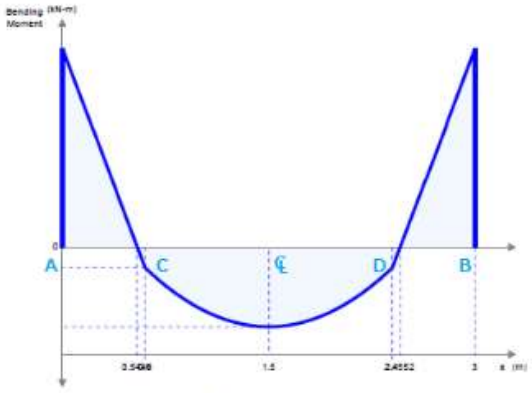
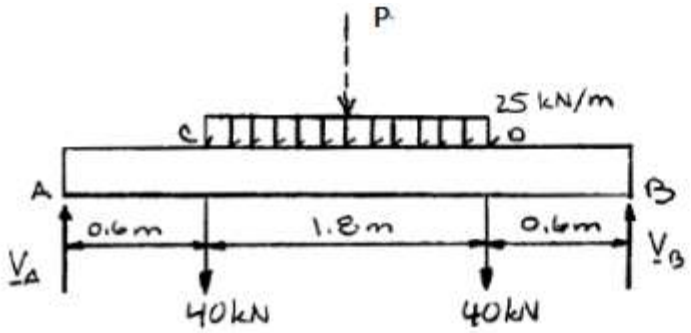


Figure 4



Reactions:

$+\circlearrowleft \Sigma M_A = 0$: $V_B(3.0\text{ m}) - (70.2\text{ kN})(1.5\text{ m}) - (40\text{ kN})(0.6\text{ m}) - (40\text{ kN})(2.4\text{ m}) = 0$

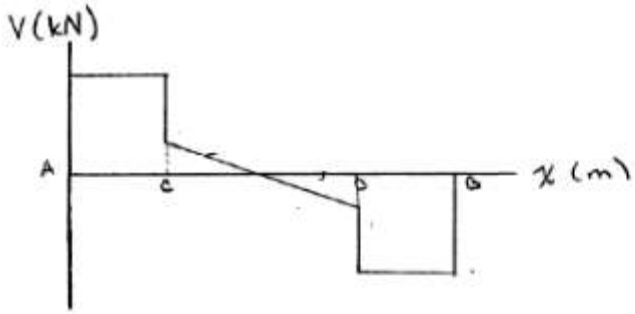
$V_B = 75.1\text{ kN}$

$+\uparrow \Sigma F_y = 0$: $V_A - 40\text{ kN} - 70.2\text{ kN} - 40\text{ kN} + 75.1\text{ kN} = 0$

$V_A = 75.1\text{ kN}$

$V_C = 35.1\text{ kN}$

$V_D = -35.1\text{ kN}$



Change in the bending moment is equal to the area under the shear diagram.

A to C: $(75.1\text{ kN})(0.6\text{ m}) = 45.06\text{ kN·m}$

C to E: $\frac{1}{2}(0.9\text{ m})(35.1\text{ kN}) = 15.79\text{ kN·m}$

E to D: $\frac{1}{2}(0.9\text{ m})(-35.1\text{ kN}) = -15.79\text{ kN·m}$

D to B: $(-75.1\text{ kN})(0.6\text{ m}) = -45.06\text{ kN·m}$

Bending moment:

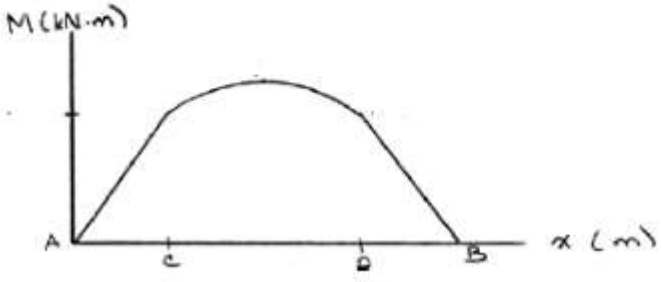
$M_A = 0$

$M_C = 0 + 45.06\text{ kN·m} = 45.06\text{ kN·m}$

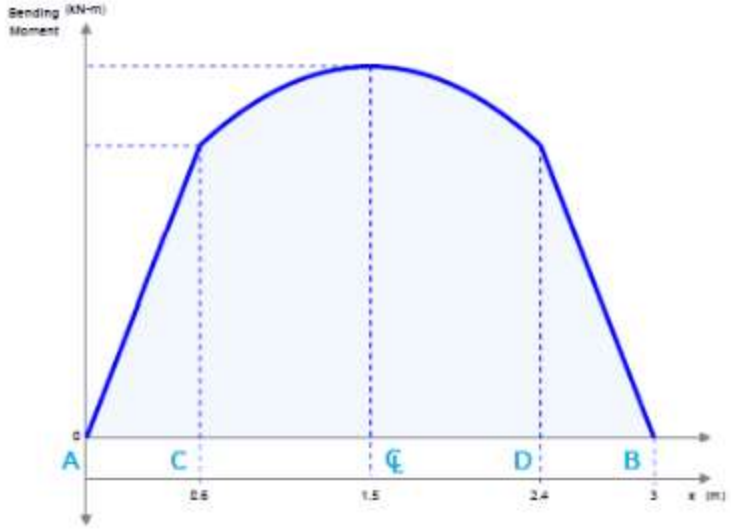
$M_E = 45.06\text{ kN·m} + 15.79\text{ kN·m} = 60.85\text{ kN·m}$

$M_D = 60.85\text{ kN·m} - 15.79\text{ kN·m} = 45.06\text{ kN·m}$

$M_B = 45.06\text{ kN·m} - 45.06\text{ kN·m} = 0$



$|M|_{\max} = 60.85\text{ kN·m}$



Problem #2:

A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490$ lb/ft^3, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 4 ksi. Take $L = 10$ ft. (Round the final answer to three decimal places.)

Let W = total weight

$$W = AL\gamma = \frac{\pi}{4}d^2L\gamma$$

Reaction at A:

$$A = \frac{1}{2}W$$

Bending moment at the center of the beam:

$$+\circlearrowleft \Sigma M_C = 0: \; -\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32}d^2L^2\gamma$$

For a circular cross section,

$$c = \frac{1}{2}d$$

$$I = \frac{\pi}{4}c^4$$

$$S = \frac{I}{c} = \frac{\pi}{4}c^3 = \frac{\pi}{32}d^3$$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32}d^2L^2\gamma}{\frac{\pi}{32}d^3} = \frac{L^2\gamma}{d}$$

Solving for d,

$$d = \frac{L^2\gamma}{\sigma}$$

Data:

$$L = 10 \text{ ft} = (12)(10 \text{ ft}) = 120 \text{ in.}$$

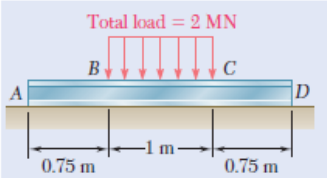
$$\gamma = 490 \text{ lb/ft}^3 = \frac{490}{12^3} = 0.28356 \text{ lb/in}^3$$

$$\sigma = 4 \text{ ksi} = 4000 \text{ lb/in}^2$$

$$d = \frac{(120 \text{ in.})^2 (0.28356 \text{ lb/in}^3)}{4000 \text{ lb/in}^2} = 1.021 \text{ in.}$$

Problem #3:

Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, determine the section modulus and select the most economical wide-flange beam to support the loading shown using the table given below. (Round the final answer to the nearest whole number.)



Shape	$S (10^3 \text{ mm}^3)$
W690 × 125	3490
W610 × 101	2520
W530 × 150	3720
W460 × 113	2390

The section modulus is 2206 ± 2% × 10^3 mm^3.

The most economical wide-flange beam is W610 x 101.

Explanation:

Downward distributed load: $w = \frac{2 \text{ MN}}{1.0 \text{ m}} = 2 \text{ MN/m}$

Upward distributed reaction: $q = \frac{2 \text{ MN}}{2.5 \text{ m}} = 0.8 \text{ MN/m}$

Net distributed load over *BC*: 1.2 MN/m

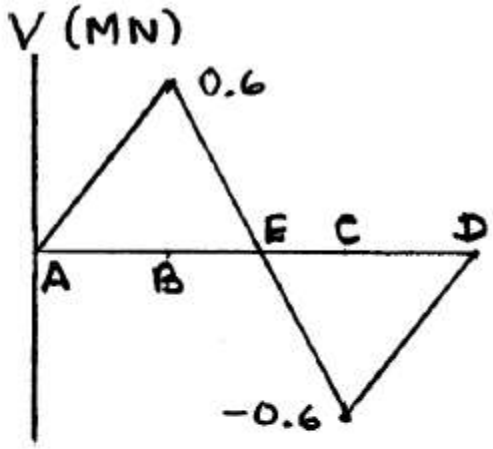
Shear:

$$V_A = 0$$

$$V_B = 0 + (0.75 \text{ m})(0.8 \text{ MN/m}) = 0.6 \text{ MN}$$

$$V_C = 0.6 \text{ MN} - (1.0 \text{ m})(1.2 \text{ MN/m}) = -0.6 \text{ MN}$$

$$V_D = -0.6 \text{ MN} + (0.75 \text{ m})(0.8 \text{ MN/m}) = 0$$



Areas:

A to B: $\left(\frac{1}{2}\right)(0.75\text{ m})(0.6\text{ MN}) = 0.225\text{ MN·m}$

B to E: $\left(\frac{1}{2}\right)(0.5\text{ m})(0.6\text{ MN}) = 0.150\text{ MN·m}$

E to C: $\left(\frac{1}{2}\right)(0.5\text{ m})(-0.6\text{ MN}) = -0.150\text{ MN·m}$

C to D: $\left(\frac{1}{2}\right)(0.75\text{ m})(-0.6\text{ MN}) = -0.225\text{ MN·m}$

Bending moments:

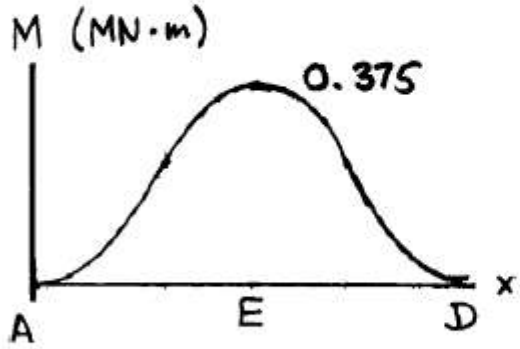
$M_A = 0$

$M_B = 0 + 0.225\text{ MN·m} = 0.225\text{ MN·m}$

$M_E = 0.225\text{ MN·m} + 0.150\text{ MN·m} = 0.375\text{ MN·m}$

$M_C = 0.375\text{ MN·m} - 0.150\text{ MN·m} = 0.225\text{ MN·m}$

$M_D = 0.225\text{ MN·m} - 0.225\text{ MN·m} = 0$



$|M|_{\max} = 0.375\text{ MN·m} = 375 \times 10^3\text{ N·m}$

$\sigma_{\text{all}} = 170\text{MPa} = 170 \times 10^6\text{ Pa}$

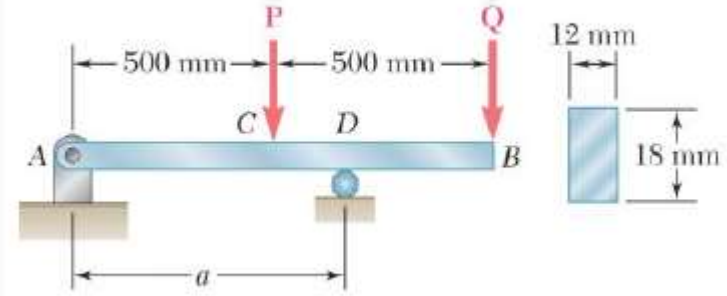
$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{375 \times 10^3\text{ N·m}}{170 \times 10^6\text{ Pa}} = 2206 \times 10^{-6}\text{ m}^3 = 2206 \times 10^3\text{ mm}^3$

Shape	$S(10^3\ \text{mm}^3)$
W690 × 125	3490
W610 × 101	2520
W530 × 150	3720
W460 × 113	2390

Lightest wide-flange section: W610 × 101

Problem #4:

Consider the given beam and loading where $P = Q = 460$ N.



Determine the distance a for which the absolute value of the bending moment in the beam is as small as possible.

The distance a is 866.03 ± 2% mm.

$P = 460$ N

$Q = 460$ N

Reaction at A:

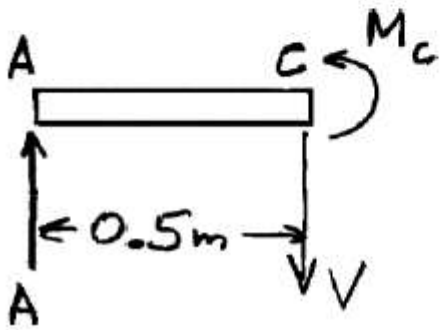
$$+\circlearrowleft \Sigma M_D = 0: \quad -Aa + 460(a - 0.5) - 460(1 - a) = 0$$

$$A = \left(920 - \frac{690}{a}\right) \text{N}$$

Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \quad -0.5A + M_C = 0$$

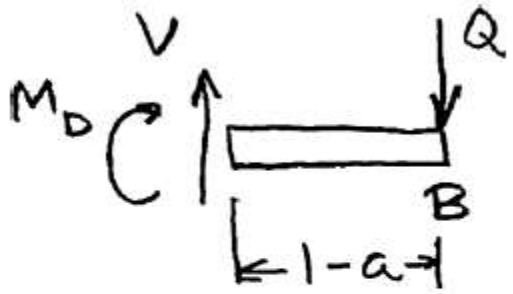
$$M_C = 0.5A = \left(460 - \frac{345}{a}\right) \text{N·m}$$



Bending moment at D:

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - 460(1-a) = 0$$

$$M_D = -460(1-a)\ \text{N·m}$$



Equate:

$$-M_D = M_C$$

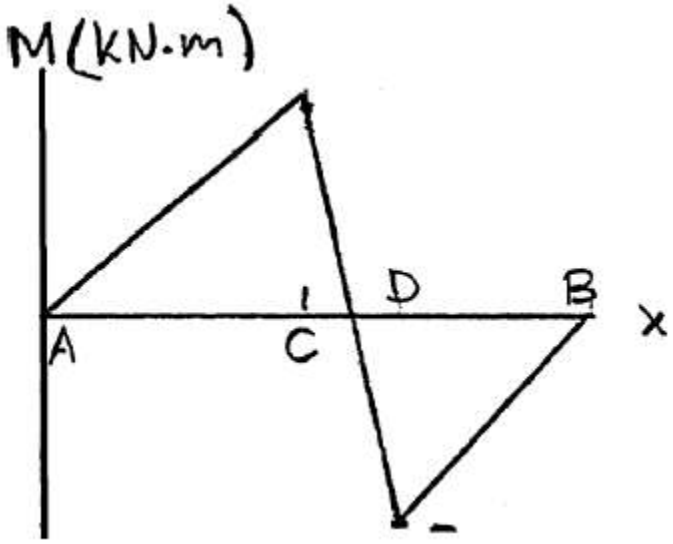
$$460(1-a) = 460 - \frac{345}{a}$$

$$a = 0.866\ \text{m} = 866.03\ \text{mm}$$

$$A = 123.26\ \text{N}$$

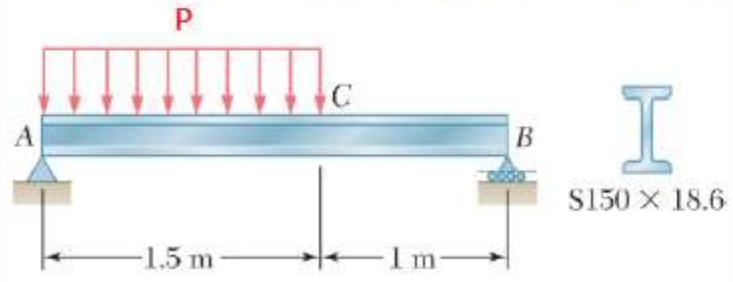
$$M_C = 61.63\ \text{N·m}$$

$$M_D = -61.63\ \text{N·m}$$



Problem #5:

Consider the given beam and loading where $P = 18$ kN/m. (For S150 × 18.6, $S = 120 \times 10^3$ mm^3.)



From the figures given below, identify the correct shear diagram. (You must provide an answer before moving to the next part.)

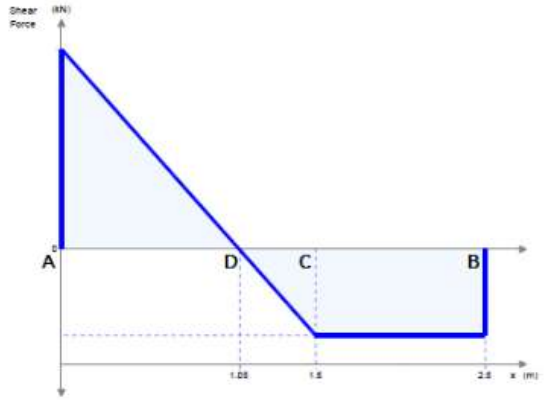


Figure 1

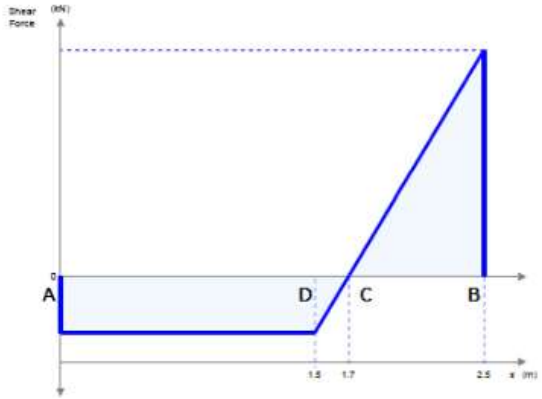


Figure 2

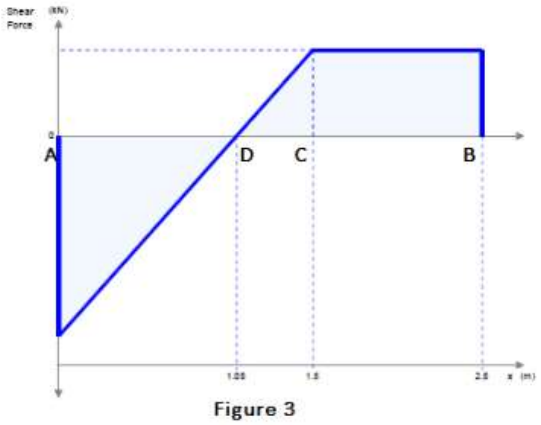


Figure 3

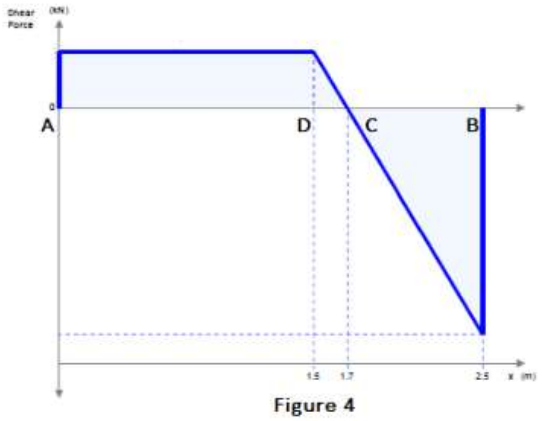


Figure 4

The correct shear diagram is Figure 1 ▾.

Explanation:

$+\circlearrowleft \Sigma M_B = 0$: $-(2.5\text{ m})A + (1.75\text{ m})(1.5\text{ m})(18\text{ kN/m}) = 0$

$A = 18.9\text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$: $-(0.75\text{ m})(1.5\text{ m})(18\text{ kN/m}) + (2.5\text{ m})B = 0$

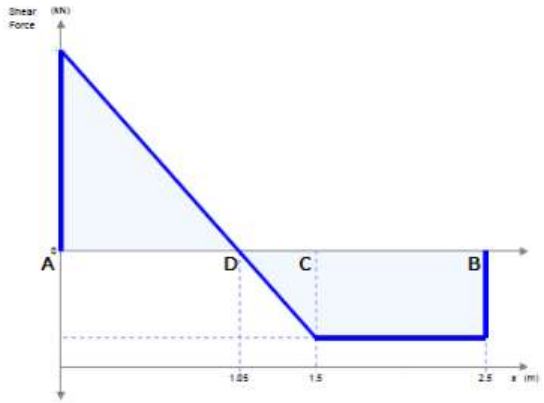
$B = 8.1\text{ kN}$

Shear diagram:

$V_A = 18.9\text{ kN}$

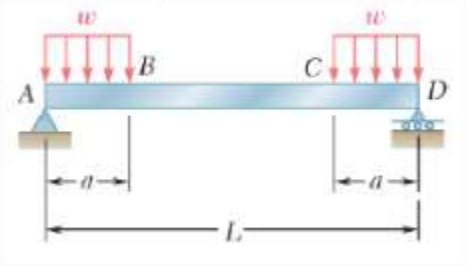
$V_C = 18.9\text{ kN} - (1.5\text{ m})(18\text{ kN/m}) = -8.1\text{ kN}$

$V_B = -8.1\text{ kN}$



Problem #6:

Consider the given beam and loading.



For the beam and loading shown, identify the equations of the shear and bending-moment curves.

- → ☐ $V = w(a - x)$
- ☐ $V = w(x - a)$
- → ☐ $V = w\{(L - x - a)\}$
- → ☐ $V = 0$
- → ☐ $M = wax - \frac{1}{2}wx^2$
- ☐ $M = wax - \frac{1}{4}wx^2$
- → ☐ $M_B = \frac{1}{2}wa^2$
- → ☐ $M = \frac{1}{2}wa^2 - w\left\{\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right\}$

Reactions: $A = D = wa$

A to B: $0 < x < a \quad w = w$

$$V_A = A = wa, \quad M_A = 0$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = w(a - x)$$

$$\frac{dM}{dx} = V = wa - wx$$

$$M - M_A = \int_0^x V\,dx = \int_0^x (wa - wx)\,dx$$

$$M = wax - \frac{1}{2}wx^2$$

$$V_B = 0$$

$$M_B = \frac{1}{2}wa^2$$

B to C: $a < x < L - a$

$$V = 0$$

$$\frac{dM}{dx} = V = 0$$

$$M - M_B = \int_0^x V\,dx = 0$$

$$M = M_B$$

$$M = \frac{1}{2}wa^2$$

C to D: $L - a < x < L$

$$V - V_C = -\int_{L-a}^{x} w\,dx = -w\{x - (L - a)\}$$

$$V = w\{(L - x - a)\}$$

$$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} \{x - (L - a)\}\,dx$$

$$= -w\left\{\frac{x^2}{2} - (L - a)x\right\}\Big|_{L-a}^{x}$$

$$= -w\left\{\frac{x^2}{2} - (L - a)x - \frac{(L-a)^2}{2} + (L - a)^2\right\}$$

$$= -w\left\{\frac{x^2}{2} - (L - a)x + \frac{(L-a)^2}{2}\right\}$$

$$M = \tfrac{1}{2}wa^2 - w\left\{\frac{x^2}{2} - (L - a)x + \frac{(L-a)^2}{2}\right\}$$